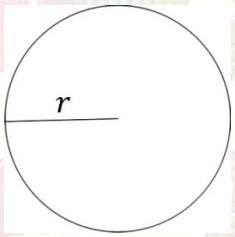
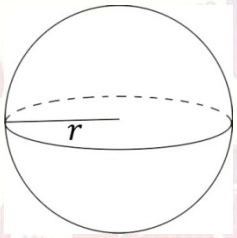
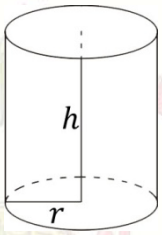
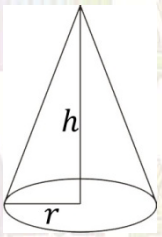
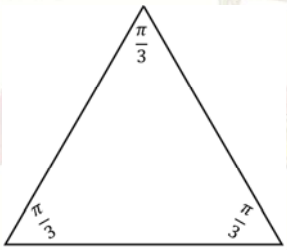
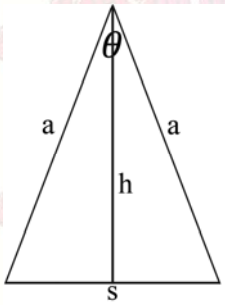
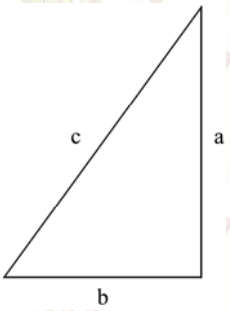
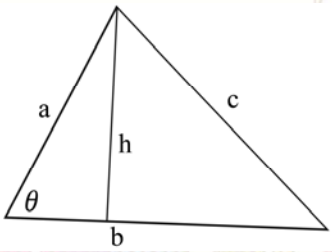
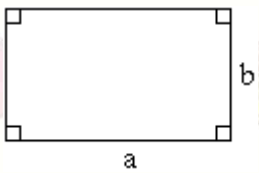
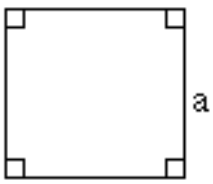
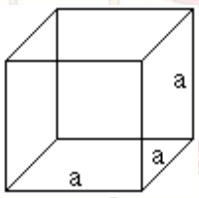
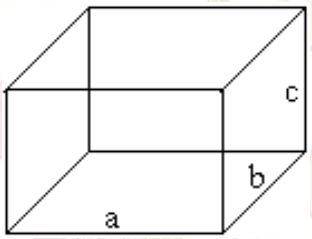
# HOSSAM GHANEM

## (26) 3.9 Related rates (A)

| Circle | Sphere | Right cylinder | Right Cone |
|---|---|---|---|
| $C = 2\pi r$ | $S = 4\pi r^2$ | $S = 2\pi rh$ | $S = 2\pi r\sqrt{r^2 + h^2}$ |
|  |  |  |  |
| $A = \pi r^2$ | $V = \frac{4}{3}\pi r^3$ | $V = \pi r^2 h$ | $V = \frac{1}{3}\pi r^2$ |

| Equal side triangle | Isosceles triangle | Right triangle | Triangle |
|---|---|---|---|
| $C = 3a$ | $C = 2a + s$ | $C = a + b + c$ | $C = a + b + c$ |
|  |  |  |  |
| $A = \frac{\sqrt{3}}{2}a^2$ | $A = \frac{1}{2}a^2 \sin\theta$<br>$A = \frac{1}{2}sh$ | $A = \frac{1}{2}ab$ | $A = \frac{1}{2}bh$<br>$A = \frac{1}{2}ab\sin\theta$ |

| Rectangle | Square | Coupe | Rectangular box |
|---|---|---|---|
| $C = 2(a + b)$ | $C = 4a$ | $S = 6a^2$ | $S = 2ab + 2ac + 2bc$ |
|  |  |  |  |
| $A = ab$ | $A = a^2$ | $V = a^3$ | $V = abc$ |

| Example 1<br>35 December 16, 2004 | Tow students start walking from the same point . One walks south at a rate $4\ m/sec$ , and the other walks west at a rate $3\ m/sec$.At what rate is distance between the two students increasing 2 seconds later |
|---|---|

## Solution

2 seconds

$x = -6 \quad , \quad y = -8 \quad , s = 10$

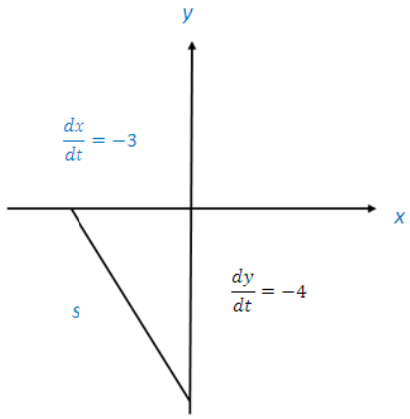
$s^2 = x^2 + y^2$

$2\,s\,\frac{ds}{dt} = 2\,x\,\frac{dx}{dt} + 2y\,\frac{dy}{dt}$

$20\,\frac{ds}{dt} = 2(-6)\,(-3) + 2\,(-8)\,(-4)$

$20\,\frac{ds}{dt} = 36 + 64$

$\frac{ds}{dt} = \frac{100}{20} = 5\ m\ /\ sec$




| Example 2<br>37 May 4, 2006 | The radius of a closed right circular cylinder is decreasing at rate of 1 $cm/sec$ and the height is increasing at a rate of 4 $cm/sec$ . Find the rate at which the total surface area of the cylinder is changing when the radius is 6 $cm$ and the height is 10 $cm$ |
|---|---|

## Solution

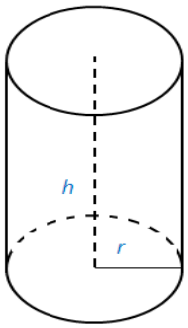
$\frac{dr}{dt} = -1 \quad , \frac{dh}{dt} = 4 \quad , r = 6 \quad , h = 10$

$s = 2\pi rh + 2\pi r^2$

$\frac{ds}{dt} = 2\pi h\,\frac{dr}{dt} + 2\pi r\,\frac{dh}{dt} + 4\pi r\,\frac{dr}{dt}$

$= 2\pi(10)(-1) + 2\pi(6)(4) + 4\pi(6)(-1)$

$= \pi\,(-20 + 48 - 24) = 4\pi$



| Example 3<br>33 May 6, 2004 | A snow ball is melting at a rate of 0.03 $ft^3/hr$. At what rate is the surface area changing when the volume of the ball is $36\pi$ ? |
|---|---|

## Solution

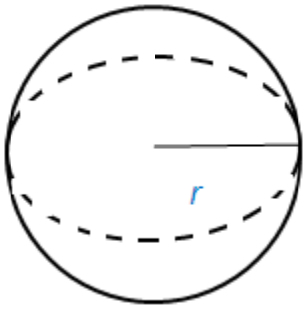
$\frac{dv}{dt} = -0.03$

$v = \frac{4}{3}\pi r^3$

$36\pi = \frac{4}{3}\pi r^3$

$r^3 = \frac{(36)\cdot(3)}{4} = (9)(3) = 27$

$r = 3$




$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

$-0.03 = 4\pi\,(9)\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{-3}{(4)(9)(100)\pi} = \frac{-1}{1200\,\pi}$

$s = 4\pi r^2$

$\frac{ds}{dt} = 8\,\pi\,r\frac{dr}{dt}$

$= 8\pi(3)\cdot\frac{-1}{1200\pi} = \frac{-1}{50}$

| Example 4<br>40 May 3, 2007 | A metal in the shape of a right circular cone, whose height is twice its radius, is being heated. If the radius is increasing at a rate of $0.001\ cm/sec$ , find the rate at which its volume is increasing when its radius is $2\ cm$ |
|---|---|

## Solution

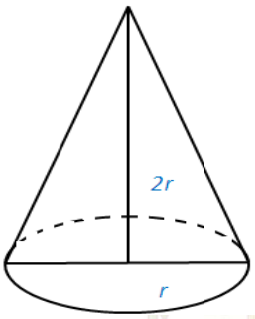
$\frac{dr}{dt} = 0.001$ , $h = 2r$ , $r = 2$

$v = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2(2r)$

$v = \frac{2}{3}\pi r^3$

$\frac{dv}{dt} = 2\pi r^2 \frac{dr}{dt}$

$= 2\pi\,(4)(0.001) = 0.008\,\pi$



| Example 5<br>38 July 17, 2006 | A plate in a shape of a disk is heated . If the area $A$ of the plate (in $cm^2$) after time $t$ ( in hours ) is given by $A = \sqrt{t^2 + 3t + 6}$<br>Find the rate at which the radius of the plate is changing after two hours |
|---|---|

## Solution

$A = \sqrt{t^2 + 3t + 6}$

$\pi r^2 = \sqrt{t^2 + 3t + 6}$

$t = 2$

$\pi r^2 = \sqrt{4 + 6 + 6}$

$r^2 = \frac{\sqrt{16}}{\pi} = \frac{4}{\pi}$

$r = \frac{2}{\sqrt{\pi}}$

$2\pi r \frac{dr}{dt} = \frac{2t + 3}{2\sqrt{t^2 + 3t + 6}}$

$2\pi \left(\frac{2}{\sqrt{\pi}}\right) \frac{dr}{dt} = \frac{4 + 3}{2(4)}$

$4\sqrt{\pi} \frac{dr}{dt} = \frac{7}{4}$

$\frac{dr}{dt} = \frac{7}{16\sqrt{\pi}}$

| Example 6<br>49 July 24, 2010 | (3 Points) A point $P(x, y)$ moves on the curve $y = \sqrt{x}$. If $S$ is the distance between the point $P$ and the origin then find $\frac{ds}{dt}$ when $x = 1$ and $\frac{dx}{dt} = 2$. |
|---|---|

## Solution

$s = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$s = \sqrt{(x - 0)^2 + (y - 0)^2}$

$s = \sqrt{x^2 + y^2}$

$s = \sqrt{x^2 + x}$

$\frac{ds}{dt} = \frac{2x + 1}{2\sqrt{x^2 + x}} \cdot \frac{dx}{dt}$

$\frac{ds}{dt} = \frac{2 + 1}{2\sqrt{1 + 1}} \cdot (2) = \frac{3}{\sqrt{2}}$

# Homework

| | |
|---|---|
| 1 | 42 May 5, 2008<br>Tow points $A$ and $B$ located at the origin ( 0 , 0 ) of the $xy$-plane. Point $A$ moves along the $x$-axis at 3 $m/min$ , and point $B$ moves along $y$-axis at 4 $m/min$. What is the rate of change of the distance between $A$ and $B$ after two minutes |
| 2 | 45 10 May, 2009<br>A car starts at a point $A$ and travels east at $80\ km/hr$ . At the same time another car starts at $A$ and goes north at $60\ km/hr$ . At what rate is the distance between them changing one hour after the cars start ? |
| 3 | 23 April 27,2000<br>A ladder 13 ft long is leaning against the side of a building. If the bottom of the ladder is pulled away from the building at a rate of $\frac{1}{10}$ ft /sec how fast is the angle formed by the ladder and the ground changing at the instant when the top of the ladder is 12 ft above the ground ? |
| 4 | 39 Dec. 14, 2006<br>A right circular cylinder is being heated, its radius is increasing at rate of 0.04 $mm/sec$ and its height is increasing at a rate of 0.15 $mm/sec$ . Find the rate at which the volume of the cylinder is changing when the radius is 0.5 $mm$ and the height is 0.3 $mm$ |
| 5 | The radius $r$ of a cylinder decreasing at a rate of *4 m/sec*. and the altitude h increasing at a rate of *4 m/sec* Does the volume increasing or decrease when $r = 5\ m$ and $h = 4\ m$ |
| 6 | 24 July 20th , 2000<br>As a right circular cylindrical metal rod is being heated, its height is increasing at a rate of 0.002 cm/ min and its radius is increasing at a rate of 0.001 cm/min. At what rate is the volume changing when the rod has height 20 cm and radius 4 cm. |
| 7 | 25 December 10, 2000<br>A right circular cylinder with closed top and bottom is being heated. Its height is increasing at a rate of 0.004 cm / min and its radius is increasing at rate of 0.002 cm/min. At what rate is the total surface area changing when the cylinder has height 20 cm and radius 4 cm? |
| 8 | 4 December 15, 1994<br>Gas is escaping from a spherical balloon at the rate of 2 $ft^3$ */min*. At what rate is the surface area decreasing when the radius is 1 ft. |
| 9 | 48 Sunday 9 May 2010<br>A meeting ends at 6.00 p.m. Amal leaves immediately, and drives North at a speed of 80 km / hour. Batool leaves a $\frac{1}{2}$ hour later and drives East at a speed of 120 km / hour How fast is the distance between them increasing at 7.00 p.m. ? |

# Homework

| | |
|---|---|
| 10 | 07/12/2011<br>(4 points ) :Two people start walking from the same point at the same time. One walks east at a rate 8 km/hr and other walks south at a rate of 6 km/ hr. How fast is the distance between them changing after half an hour ? |